



Woodlands Federation

White
Rose
Maths



Policy for written calculations - Mathematics

Date: February 2018

Document summary

This document sets out the agreed progression from mental methods through to standard formal written calculations.

At all stages it is important that teaching is supported with structured models and images so that concepts can be embedded. This will be implemented by all age groups using concrete, pictorial and abstract within mathematics teaching.

It is more important that pupils understand what they are doing and can apply their 'method' to a given situation, than it is simply to be able to complete a standard written method.

We also agree to use precise mathematical vocabulary and encourage children to do the same.


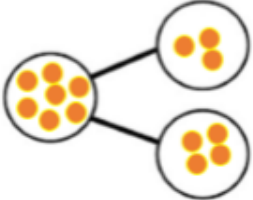
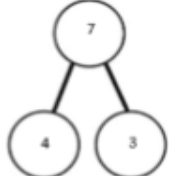
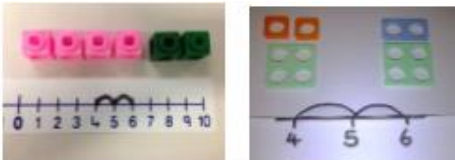
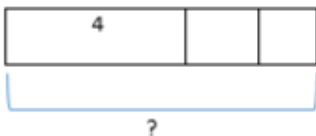

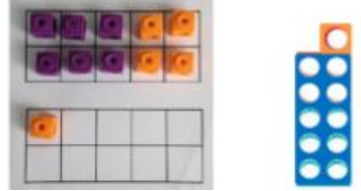

We know that it is important to offer calculations in a range of representations. This is to include part, part whole; bar modelling; the tens frame; missing numbers and expressions where the = sign is not always in the traditional place.

This document should be read in conjunction with the 'Mathematics Policy'.

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole; part whole model. Starting at the bigger number and counting on. Regrouping to make 10.	Adding three single digits. Column method – no regrouping.	Column method- regrouping. (up to 3 digits)	Column method- regrouping. (up to 4 digits)	Column method- regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method- regrouping. (Decimals- with different amounts of decimal places)
Subtraction	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Make 10 Column method- no regrouping	Column method with regrouping. (up to 3 digits)	Column method with regrouping. (up to 4 digits)	Column method with regrouping. (with more than 4 digits) (Decimals- with the same amount of decimal places)	Column method with regrouping. (Decimals- with different amounts of decimal places)
Multiplication	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Repeated addition Arrays- showing commutative multiplication	Counting in multiples Repeated addition Arrays- showing commutative multiplication Grid method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (multi digit up to 4 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping	Division as grouping Division within arrays	Division within arrays Division with a remainder Short division (2 digits by 1 digit- concrete and pictorial)	Division within arrays Division with a remainder Short division (up to 3 digits by 1 digit- concrete and pictorial)	Short division (up to 4 digits by a 1 digit number interpret remainders appropriately for the context)	Short division Long division (up to 4 digits by a 2 digit number- interpret remainders as whole numbers, fractions or round)

CALCULATION POLICY FOR ADDITION

Key language which should be used: sum, total, parts and wholes, plus, add, altogether, more than, 'is equal to' 'is the same as'.

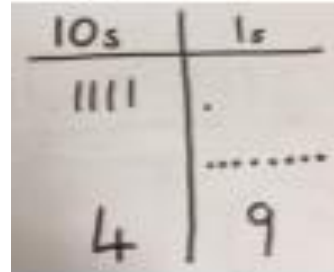
Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole (use other resources too e.g. shells, teddy bears etc.)</p> 	<p>Children to represent the counters using dots.</p> 	<p>$4 + 3 = 7$ (four is a part, 3 is a part and the whole is seven.)</p> 
<p>Counting on using number lines by using cubes or Numicon.</p> 	<p>A bar model which encourages the children to count on rather than count all.</p> 	<p>The abstract number line: What is 2 more than 4? What is the sum of 2 and 4? What is the total of 4 and 2? $4+2$</p> 
<p>Regrouping to make 10; using ten frames. $6+5$</p> 	<p>Children to draw the tens frame.</p> 	<p>Children to develop an understanding of equality. e.g.</p> <p>$6 + \square = 11$ $6 + 5 = 5 + \square$ $6 + 5 = \square + 4$</p>

TO+O; using base 10 to develop understanding of partitioning and place value.

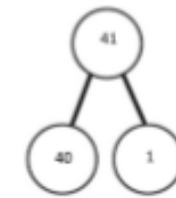
41+8 =



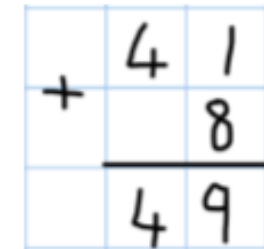
Children to represent the base 10. e.g. lines for tens and dots for ones.



= 41+8



$1 + 8 = 9$
 $40 + 9 = 49$

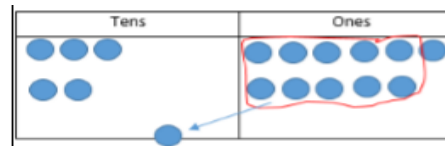
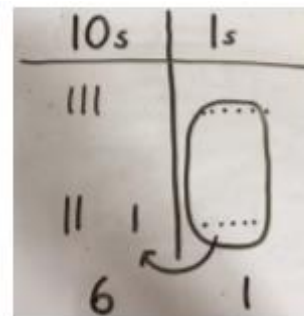


TO+TO; using base 10 to develop understanding of partitioning and place value.

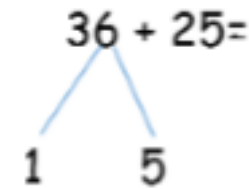
36+25

	Tens	Ones
+		
=		

Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

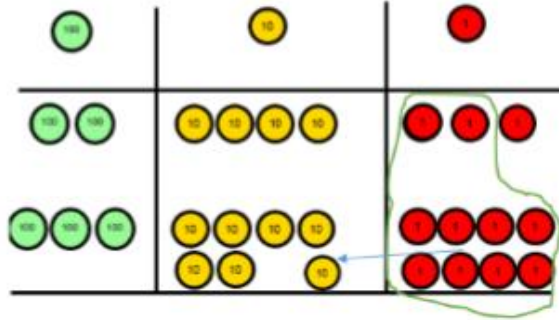


$30 + 20 = 50$
 $5 + 5 = 10$
 $50 + 10 + 1 = 61$

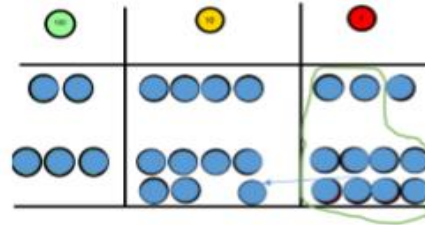
Formal method:

$$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$$

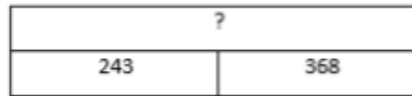
Use of place value counters to add HTO+TO, HTO+HTO etc. When there are 10 ones in the 1s column- we exchange for 1 ten; when there is 10 tens in the 10s column - we exchange for 1 hundred.



Children to represent the counters in a place value chart, circling when they make an exchange.

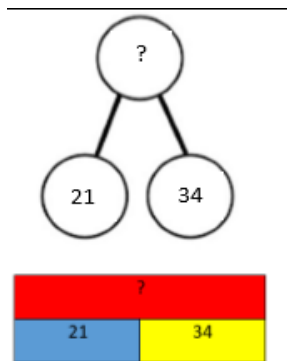


If the children are completing a word problem, draw a bar model to represent what it's asking them to do.



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ \hline 1 \quad 1 \end{array}$$

Conceptual Variation, different ways to ask children to solve 21+34



Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

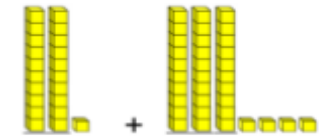
$21 + 34 = 55$. Prove it

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$21 + 34 =$

$\square = 21 + 34$

What's the sum of twenty one and thirty four?



Always use missing digit problems too:

Tens	Ones
● ●	●
● ● ●	?
?	4

MENTAL CALCULATIONS FOR ADDITION

These are a **selection** of mental calculation strategies:

It is very important that children know and can use their number facts.

Mental recall of number bonds

$$6 + 4 = 10 \quad \square + 3 = 10 \quad 25 + 75 = 100 \quad 19 + \square = 20$$

Use near doubles

$$6 + 7 = \text{double } 6 + 1 = 13$$

Addition using partitioning and recombining

$$34 + 45 = (30 + 40) + (4 + 5) = 79$$

Counting on or back in repeated steps of 1, 10, 100, 1000

$$86 + 57 = 143 \text{ (by counting on in tens and then in ones)} \quad 460 - 300 = 160 \text{ (by counting back in hundreds)}$$

Add the nearest multiple of 10, 100 and 1000 and adjust

$$24 + 19 = 24 + 20 - 1 = 43 \quad 458 + 71 = 458 + 70 + 1 = 529$$

Use the relationship between addition and subtraction

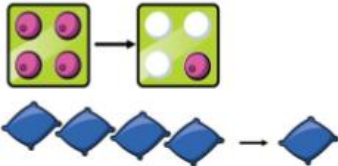
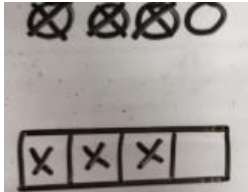
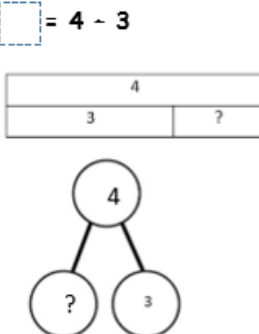
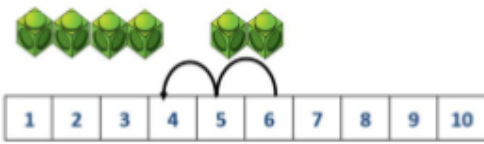
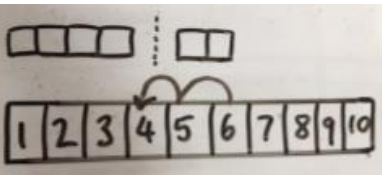
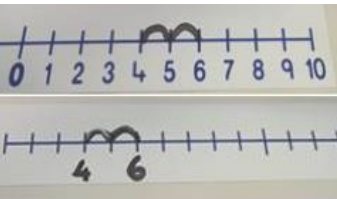
$$36 + 19 = 55 \quad 19 + 36 = 55$$

$$55 - 19 = 36 \quad 55 - 36 = 19$$

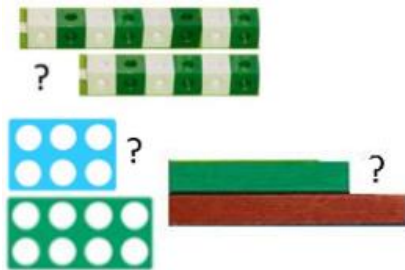
MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

CALCULATION POLICY FOR SUBTRACTION

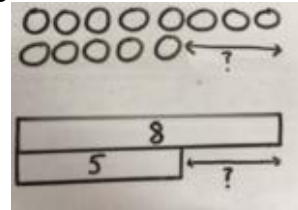
Key language which should be used: take away, less than, the difference, subtract, minus, fewer, decrease.

Concrete	Pictorial	Abstract
<p>Physically taking away and removing objects from a whole.</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out. The bar model can also be used.</p> 	<p>$4 - 3 = 1$</p> <p><input type="text"/> = $4 - 3$</p> 
<p>Counting back (using number line or number tracks) Children to start with 6 and count back 2.</p> 	<p>Children to represent what they see pictorially.</p> 	<p>Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.</p> 

Finding the difference;



Children to draw the counters/other concrete objects which they have used. The children should be encouraged to use the bar model.



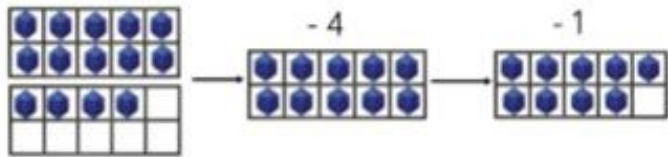
Find the difference between 8 and 5.

8 - 5, the difference is

Children to
 $14 - 5 = 9$
 $9 - 6 = 8 -$
 difference. $4 \quad 1$ ve the same

Making 10; using the tens frames.

14-5 =

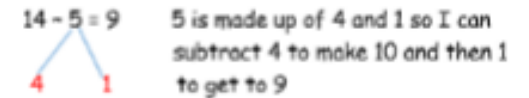
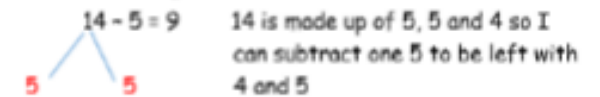


Children to present the tens frame pictorially and discuss what they did to make 10.



14 - 5 = 9 You also want children to see related facts e.g. 15 - 9 = 5

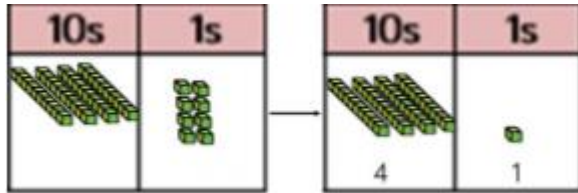
Children to represent how they have solved it e.g.



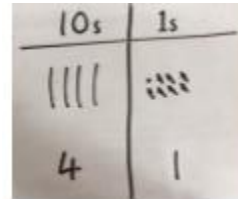
Children to show how they can make 10 by partitioning.

Column method; using base 10.

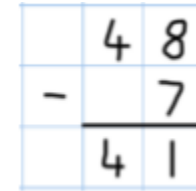
48-7



Children to represent the base 10 pictorially.



Column method or children could count back 7.



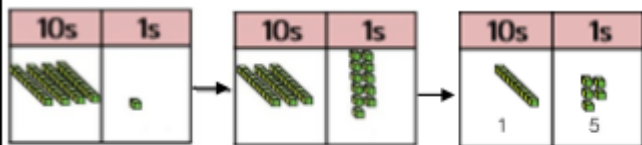
Column method; using base 10 and having to exchange.

45-26

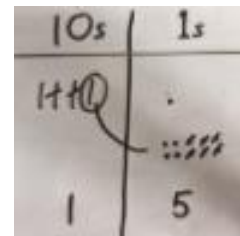
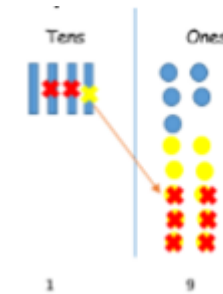


- 1) Start by partitioning 45
- 2) Exchange one ten for ten more ones
- 3) Subtract the ones, then the tens.

41-26



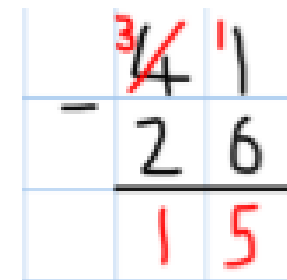
Children to represent the base 10 pictorially, remembering to show the exchange.



Formal column method. Children must understand that when they have exchanged the 10 they still have:

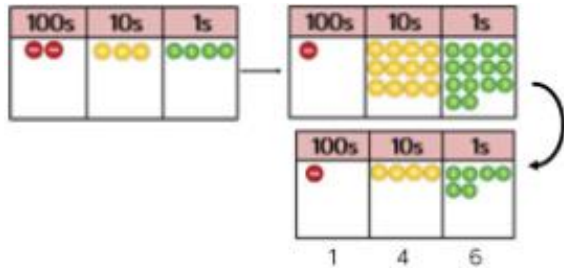
$$45 = 30 + 15$$

$$41 = 30 + 11.$$

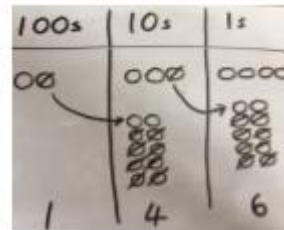


Column method; using place value counters.

234-88



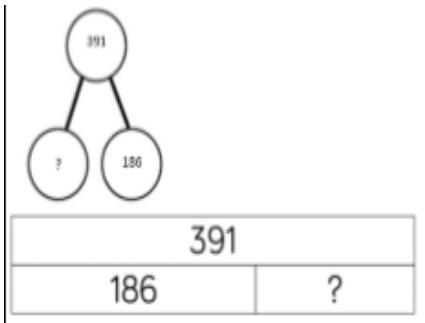
Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} 234 \\ - 88 \\ \hline 6 \end{array}$$

Conceptual Variation, different ways to ask children to solve 391-186



Raj spent £391, Timmy spent £186. How much more did Raj spend?

Calculate the difference between 391 and 186.

$$\square = 391 - 186$$

$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

What is 186 less than 391?

Missing digit calculations

$$\begin{array}{r} 39\square \\ - \square\square 6 \\ \hline \square 0 5 \end{array}$$

What's the calculation? What's the answer?



MENTAL CALCULATIONS FOR SUBTRACTION

These are a **selection** of mental calculation strategies:

It is important that children know and can apply their number facts.

Mental recall of addition and subtraction facts

$10 - 6 = 4$

$17 - \square = 11$

$20 - 17 = 3$

$10 - \square = 2$

Find a small difference by counting up

$82 - 79 = 3$

Counting on or back in repeated steps of 1, 10, 100, 1000

$86 - 52 = 34$ (by counting back in tens and then in ones)

$460 - 300 = 160$ (by counting back in hundreds)

Subtract the nearest multiple of 10, 100 and 1000 and adjust

$24 - 19 = 24 - 20 + 1 = 5$

$458 - 71 = 458 - 70 - 1 = 387$

Use the relationship between addition and subtraction

$36 + 19 = 55$

$19 + 36 = 55$


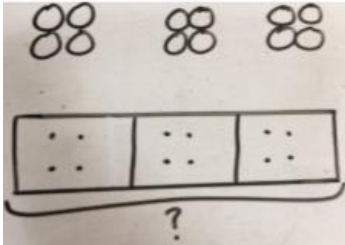
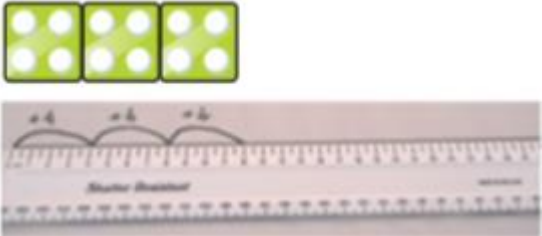
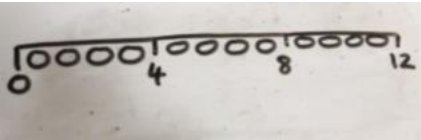
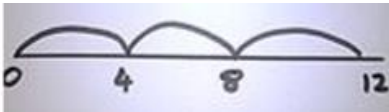
$55 - 19 = 36$

$55 - 36 = 19$

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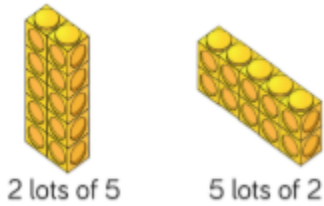
CALCULATION POLICY FOR MULTIPLICATION

Key language which should be used: double times, multiplied by, the product of, groups of, lots of.

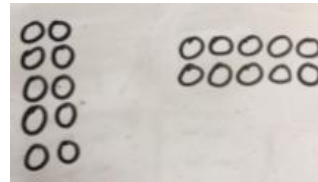
Concrete	Pictorial	Abstract
<p>Repeated grouping/repeated addition (does not have to be restricted to cubes)</p> <p>3×4</p> <p>There are 3 equal groups, with 4 in each group.</p> <p>$4+4+4$</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$</p> <p>$12 = 3 \times 4$</p> <p>$4+4+4 = 12$</p>
<p>Number Lines to show repeated groups.</p> <p>3×4</p>  <p>Cuisenaire rods can be used too.</p>	<p>Represent this pictorially alongside a number line e.g.</p> 	<p>Abstract number line showing three jumps of four.</p> <p>$3 \times 4 = 12$</p> 

Use arrays to illustrate commutativity;
(counters and other objects can also be used)

$$2 \times 5 = 5 \times 2$$



Children to represent the arrays pictorially.



Children to be able to use an array to write a range of calculations e.g.

$$10 = 2 \times 5$$

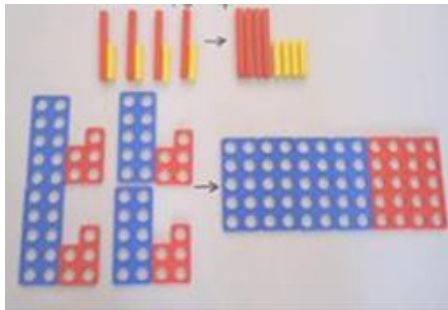
$$5 \times 2 = 10$$

$$2 + 2 + 2 + 2 + 2 = 10$$

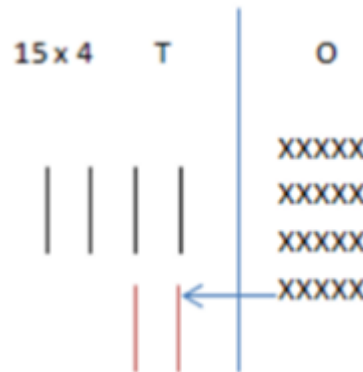
$$10 = 5 + 5$$

Partition to multiply; using Numicon, base 10 or Cuisenaire rods.

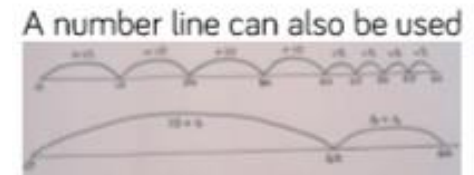
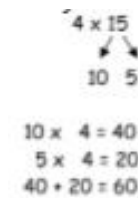
$$15 \times 4$$



Children to represent the concrete manipulatives pictorially.

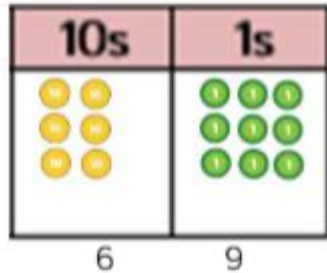


Children to be encouraged to show the steps they have taken.

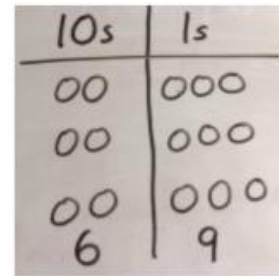


Formal Column Method; with place value counters (base ten can also be used)

$$3 \times 23$$



Children to represent the counters pictorially.

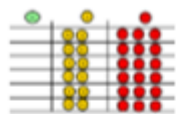


Children to record what it is they are doing to show understanding.

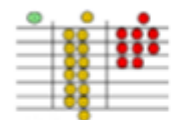
$$\begin{array}{r}
 3 \times 23 \\
 \swarrow \searrow \\
 20 \quad 3 \\
 3 \times 20 = 60 \\
 3 \times 3 = 9 \\
 60 + 9 = 69 \\
 \\
 23 \\
 \times 3 \\
 \hline
 69
 \end{array}$$

Formal Column Method with place value counters.

$$6 \times 23$$



Step 1: get 6 lots of 23



Step 2: 6×3 is 18. Can I make an exchange? Yes! Ten ones for one ten...

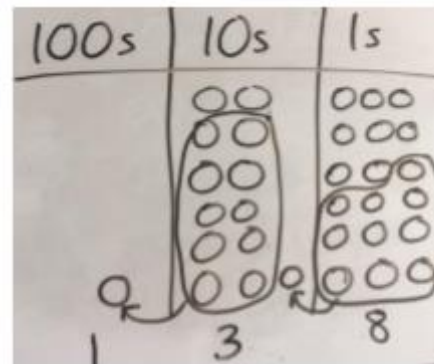


Step 3: 6×2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred...



Step 4- what do I have I each column?

Children to represent the counters/ base 10 pictorially.



$$6 \times 23 =$$

$$\begin{array}{r}
 23 \\
 \times 6 \\
 \hline
 138 \\
 \hline
 1 \quad 1
 \end{array}$$

When children start to multiply 3digit x 3 digit and 4 digit x 2 digit ... they should be confident with the abstract:

To get 744 children have solved 6×124

To get 2480 they have solved 20×124

$$\begin{array}{r}
 \text{1 2 4} \\
 \times \quad \text{2 6} \\
 \hline
 \text{7 4 4} \\
 \text{2 4 8 0} \\
 \hline
 \text{3 2 2 4} \\
 \hline
 \text{1 1}
 \end{array}$$

Answer: 3224

Conceptual Variation, different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

With the counters, prove that

$$6 \times 23 = 138$$

Explain why...

$$6 \times 23 = 23 \times 6$$

Mai had to swim 23 lengths, 6 times a week. How many lengths did she swim in one week?

Tom saved 23p three days a week. How much did he save in 2 weeks?

Find the product of 6 and 23.



$$6 \times 23 =$$

$$\square = 6 \times 23$$

$$\begin{array}{r}
 6 \quad 23 \\
 \times \quad \underline{23} \quad \times \quad \underline{6} \\
 \hline
 \hline
 \end{array}$$

What is the calculation?

What is the product?

100s	10s	1s
		

MENTAL CALCULATIONS FOR MULTIPLICATION

These are a **selection** of mental calculation strategies:

Doubling and halving

Applying the knowledge of doubles and halves to known facts. e.g. 8×4 is double 4×4

Using multiplication facts

Tables should be taught every day from Y1 onwards, either as part of the mental oral starter or other times as appropriate within the day. **Pupils to be confident using the counting stick and able to recall and use multiplication and division facts.**

Year 1	x2, x5 and x10 tables
Year 2	x2, x5, x10 and x3 tables
Year 3	x3, x6, x4 and x8 tables
Year 4	all tables up to 12×12

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts. e.g. If I know $3 \times 7 = 21$, what else do I know?
 $30 \times 7 = 210$, $300 \times 7 = 2100$, $3000 \times 7 = 21\ 000$, $0.3 \times 7 = 2.1$ etc

Use closely related facts already known

$$13 \times 11 = (13 \times 10) + (13 \times 1) = 130 + 13 = 143$$

Multiplying by 10 or 100

Knowing that the effect of multiplying by 10 is a shift in the digits one place to the left.

Knowing that the effect of multiplying by 100 is a shift in the digits two places to the left.

Partitioning

$$23 \times 4 = (20 \times 4) + (3 \times 4) = 80 + 12 = 102$$

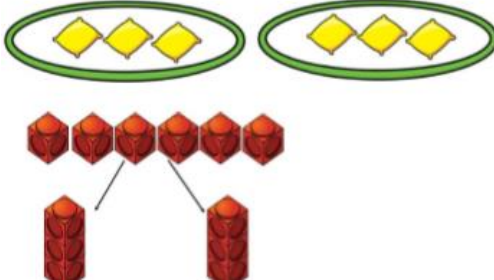
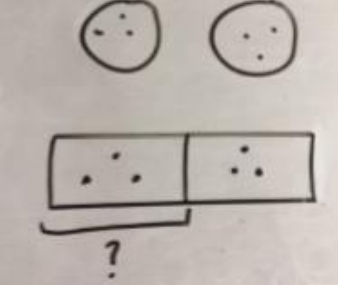

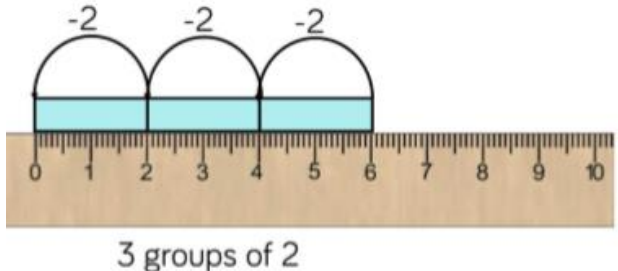
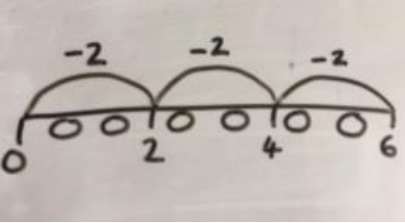
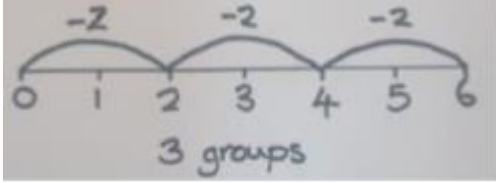
Use of factors

$$8 \times 12 = 8 \times 4 \times 3$$

MANY MENTAL CALCULATION STRATEGIES WILL CONTINUE TO BE USED. THEY ARE NOT REPLACED BY WRITTEN METHODS.

CALCULATION POLICY FOR DIVISION

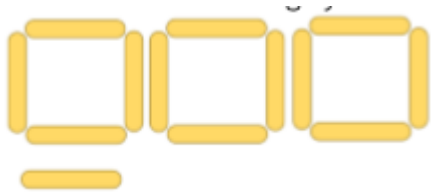
Key language which should be used: share, group, divide, divided by, half.

Concrete	Pictorial	Abstract
<p>Sharing using a range of objects. $6 \div 2$</p>  <p>The image shows 6 yellow diamonds arranged in two groups of three, each group enclosed in a green oval. Below this, there are 6 red cubes arranged in a single row. Two arrows point from the first and fourth cubes to two separate vertical stacks of three cubes each, illustrating the sharing process.</p>	<p>Represent the sharing pictorially.</p>  <p>The image shows two hand-drawn circles, each containing three dots. Below them is a hand-drawn rectangle divided into two equal halves, with three dots in each half. A bracket underneath the first half is labeled with a question mark, representing the unknown number of groups.</p>	<p>$6 \div 2 = 3$</p>  <p>The image shows a simple number line with two boxes, each containing the number 3, representing the result of the division.</p>
<p>Repeated subtraction using Cuisenaire rods. $6 \div 2$</p>  <p>The image shows a wooden ruler with a blue Cuisenaire rod placed on it. Three arches are drawn above the rod, each labeled '-2', starting from 0 and ending at 2, 4, and 6. Below the ruler, the text '3 groups of 2' is written.</p>	<p>Children to represent repeated subtraction pictorially.</p>  <p>The image shows a hand-drawn number line from 0 to 6 with circles at each integer. Three arches are drawn above the line, each labeled '-2', starting from 0 and ending at 2, 4, and 6.</p>	<p>Abstract number line to represent the equal groups that have been subtracted.</p>  <p>The image shows a hand-drawn number line from 0 to 6 with circles at each integer. Three arches are drawn above the line, each labeled '-2', starting from 0 and ending at 2, 4, and 6. Below the line, the text '3 groups' is written.</p>

2 digit ÷ 1 digit with remainders; using lollipop sticks or Cuisenaire rods.

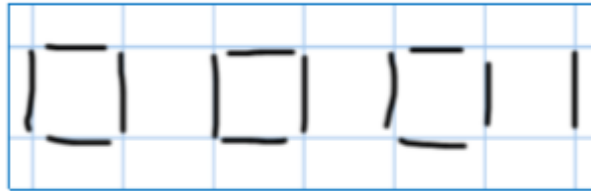
$$13 \div 4$$

Using lollipop sticks to form wholes - squares are made because we are dividing by 4.

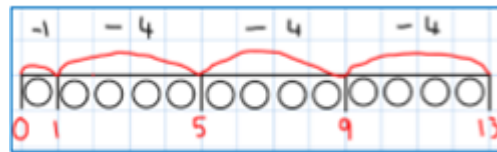


There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.



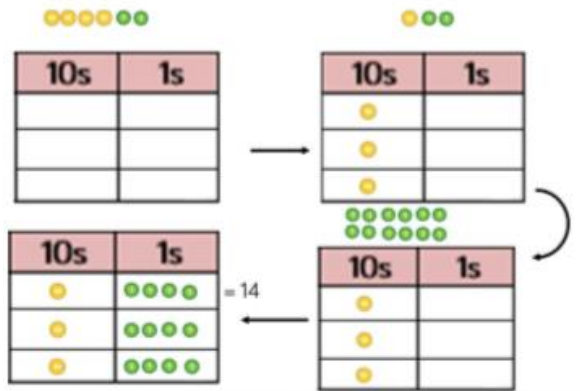
There are 3 whole squares, with 1 left over.



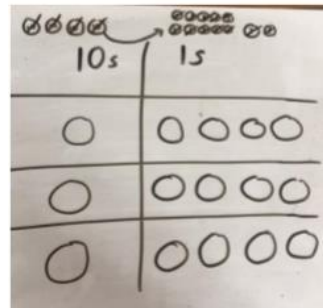
$$13 \div 4 = 3 \text{ remainder } 1$$

Sharing using place value counters;

$$42 \div 3 = 14$$



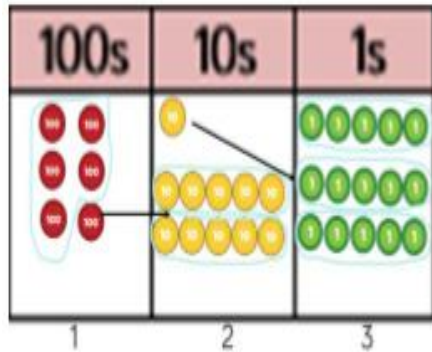
Children to represent the place value counters pictorially.



$$\begin{aligned} 42 \div 3 \\ 42 &= 30 + 12 \\ 30 \div 3 &= 10 \\ 12 \div 3 &= 4 \\ 10 + 4 &= 14 \end{aligned}$$

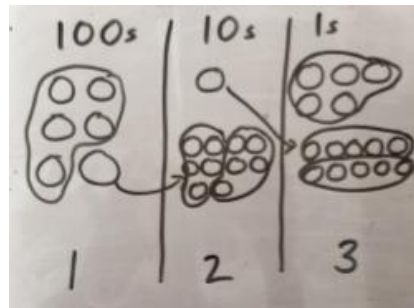
Short division (Bus Stop); with place value counters to group.

$$615 \div 5$$



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Children to represent the counters pictorially.



Children to record the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \end{array}$$

Conceptual Variation, different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$




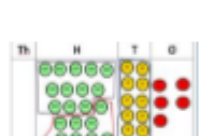

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?

100s	10s	1s

Long Division:

Concrete	Pictorial	Abstract
 <p>$2544 \div 12$ How many groups of 12 thousands do we have? None</p>	<p>Children to represent the counters, pictorially and record the subtractions beneath.</p>	<p>Step one- exchange 2 thousand for 20 hundreds so we now have 25 hundreds.</p> $12 \overline{) 2544}$
 <p>Exchange 2 thousand for 20 hundreds.</p>		<p>Step two- How many groups of 12 can I make with 25 hundreds? The 24 shows the hundreds we have grouped. The one is how many hundreds we have left.</p> $12 \overline{) 2544}$ $\underline{24}$ 1
 <p>How many groups of 12 are in 25 hundreds? 2 groups. Circle them.</p>		<p>Exchange the one hundred for 10 tens. How many groups of 12 can I make with 14 tens?</p> $12 \overline{) 2544}$ $\underline{24}$ 14 $\underline{12}$ 2
<p>We have grouped 24 hundreds so can take them off and we are left with one.</p>		<p>The 14 shows how many tens I have, the 12 is how many I grouped and the 2 is how many I have left.</p>
 <p>Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2.</p>		<p>Exchange the 2 tens for 20 ones. The 24 is how many ones I have grouped and the 0 is what I have left.</p> $12 \overline{) 2544}$ $\underline{24}$ 14 $\underline{12}$ 24 $\underline{24}$ 0
 <p>Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2</p>		

MENTAL CALCULATIONS FOR DIVISION

These are a **selection** of mental calculation strategies:

Doubling and halving

Knowing that halving is dividing by 2

Deriving and recalling division facts

Tables should be taught every day from Y1 onwards, either as part of the mental oral starter or other times as appropriate within the day. **Pupils to be confident using the counting stick and able to recall and use multiplication and division facts.**

Using and applying division facts

Children should be able to utilise their tables knowledge to derive other facts. e.g. If I know $3 \times 7 = 21$, what else do I know?

$$21 \div 3 = 7 \qquad 21 \div 7 = 3 \qquad 3 = 21 \div 7 \qquad \text{and} \qquad 7 = 21 \div 3$$

Dividing by 10 or 100

Knowing that the effect of dividing by 10 is a shift in the digits one place to the right.

Knowing that the effect of dividing by 100 is a shift in the digits two places to the right.

Use of factors

$$378 \div 21 \qquad 378 \div 3 = 126 \qquad 378 \div 21 = 18 \qquad 126 \div 7 = 18$$

Use related facts

Given that $1.4 \times 1.1 = 1.54$

What is $1.54 \div 1.4$, or $1.54 \div 1.1$?